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# Non-linear electrical conductivity and sliding in a two-dimensional electron crystal on liquid helium

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**Abstract.** At a sufficiently low temperature electrons on the surface of liquid helium form a classical Wigner crystal. As long as the electron drift velocity is not too large the electron lattice distorts the helium surface to form a lattice of dimples, which leads to an enhanced electron effective mass. This system is observed to exhibit a complicated non-linear magnetoconductivity, which we discuss in terms of a simple classical model. The model displays in a unified way the effects of two phenomena that have been treated separately by other authors: Bragg–Çerenkov scattering of capillary waves, and decoupling of the electrons from the dimples at high electric fields (sliding). Our results differ in important ways from those of the other authors.

## 1. Introduction

Electrons can be trapped in surface states above the free surface of liquid helium [1]. A sheet of such electrons forms a classical Wigner crystal when the plasma parameter, $\Gamma = e^2 n_0^{1/2}/4\pi^{1/2}\varepsilon_0 k_B T$, exceeds a value equal to about 127 [2]. For a typical electron areal density, $n_0$, of $10^{12}$ m$^{-2}$, the melting temperature is about 200 mK. As long as the crystal is not exposed to too large a horizontal (driving) electric field the electrons distort the surface of the helium to form a lattice of dimples, which has the effect of increasing the effective mass of the electrons [3]. In spite of the fact that electrons form such a simple system, magnetoresistance experiments on the crystal phase reveal a very complex non-linear behaviour at rather large driving electric fields, which we shall discuss in this paper.

Experiments generally make use of a Corbino geometry, in which the response of the system is measured when an ac drive voltage, $V$, is applied between concentric inner and outer electrodes, the magnetic field, $B$, being applied normal to the electron sheet. The electrodes couple to the electrons through capacitances that have impedances much larger than that of the electron system, so that $V$ determines (and is proportional to) the radial component of the current density in this system. The electric field in a Corbino geometry must be purely radial. Figure 1 shows schematically how the magnetoconductivity, $\sigma_{xx}$, varies with $V$ ($x$ is radial), according to Shirahama and Kono [4]; the magnetic field is large enough to ensure that $\omega_c\tau \gg 1$, where $\omega_c$ is the cyclotron frequency and $\tau$ is the electron relaxation time, so that the Hall angle is close to $\pi/2$. We distinguish three regions that show different behaviour as $V$ increases. At small values of $V$ there is a region, I, in which $\sigma_{xx}^{-1}$ rises rapidly from a low value to a maximum; for intermediate values of $V$ there is a region, II, in which $\sigma_{xx}^{-1}$ falls, approximately as $V^{-1}$; in a third region, III, at high $V$, $\sigma_{xx}^{-1}$ suddenly increases, in a hysteretic manner, to a value close to that expected in the fluid phase, in which the electron effective mass has not been enhanced by the formation of dimples, and in which the conductivity is almost

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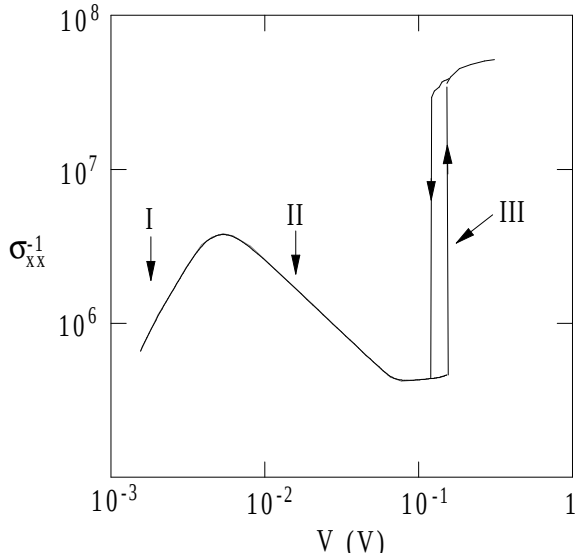


**Figure 1.** A schematic plot showing how, typically, the magnetoconductivity, $\sigma_{xx}$, varies with the driving voltage, $V$, in the experiments of Shirahama and Kono [4].

independent of $V$. Similar results have been reported by Lea's group (see, for example, [5]), although region I is seen only occasionally.

According to Shirahama and Kono the sharp rise in $\sigma_{xx}^{-1}$ in region III is due to a decoupling of the electron crystal from the dimple lattice (sliding) when the electric field is sufficiently high, with a consequent disappearance of the dimple lattice. The non-linear behaviour in regions I and II is taken into account by Shirahama and Kono but is not explained. An explanation of the behaviour in region II has been given by Dykman and Rubo [6]. They attribute the behaviour in this region to an azimuthal electron (Hall) velocity that is limited by 'Bragg–Çerenkov' scattering of capillary waves to a constant value equal to the phase velocity of capillary waves on the helium surface with wavevector equal in magnitude to the smallest reciprocal lattice vector, $\boldsymbol{G}_1$, in the electron crystal (we refer to the simplest case when the electron velocity is parallel to $\boldsymbol{G}_1$). Region I remains a mystery, and it seems to call for further experimental investigation.

It must be the case that the electron–dimple decoupling and the Bragg–Çerenkov scattering are interconnected, and this should be taken into account in a proper treatment of the decoupling. In this paper we suggest such a treatment, albeit with some simplifications. In contrast to the work of Dykman and Rubo, our treatment is entirely classical; the essential physics seems to be contained in such a treatment. Our results substantiate a suggestion by Fisher *et al* [3] that decoupling is associated with a movement of the electron crystal at a velocity exceeding the phase velocity of capillary waves with wavevector equal to $\boldsymbol{G}_1$.

## 2. Analysis of the classical model

For the present we shall ignore the Corbino geometry: we shall take the electric field as pointing uniformly in the $x$-direction in Cartesian coordinates; the magnetic field is in the $z$-direction, which we take to be away from the surface, so that the electron drift motion is mostly in the $y$-direction (large Hall angle). We shall base our discussion on a simple one-dimensional model

that focuses on the electron motion in the $y$-direction; the correct two-dimensional model is easily developed and leads to no new physics. The existence of the electron crystal leads to a steady vertical force acting on the liquid surface, with Fourier components at the reciprocal lattice vectors of the crystal, and it is this force that is responsible for the dimples. We shall suppose for simplicity that the smallest reciprocal lattice vector, $\boldsymbol{G}_1$, points in the $y$-direction, and we shall take into account distortion of the surface due only to this Fourier component of the vertical force (again a more accurate model is easily developed). This component has the form

$$f = n_0 e E_z \mathcal{R}[\exp \mathrm{i}(G_1 y - \Omega t)] \tag{1}$$

where $E_z$ is the vertical electric holding field (including the image field), and $\mathcal{R}$ denotes the real part. Electrons are at the points given by $G_1 y = \Omega t + 2n\pi$, where $n$ is an integer, so that the electron crystal is moving with steady velocity $v_y = \Omega/G_1$ in the $y$-direction. Because the Hall angle is large $v_y \gg v_x$. We have assumed that the Debye–Waller factor for the crystal is unity; the necessary correction is easy to make (see [4]). To find the corresponding distortion of the helium surface we note that the boundary condition at the free surface of the liquid ($z = 0$) can be written in the form [7]

$$-\frac{\partial f}{\partial t} + \rho \frac{\partial^2 \phi}{\partial t^2} - \alpha \frac{\partial}{\partial z}\left(\frac{\partial^2 \phi}{\partial y^2}\right) = 0 \qquad \text{at } z = 0 \tag{2}$$

where $\phi$ is the velocity potential describing the motion of the helium, which has density $\rho$ and surface tension $\alpha$. Neglect of the effect of gravity in equation (2) is justified at the relevant frequencies. We take $\phi$ to have the form

$$\phi = \mathcal{R}[\phi_0 \exp(G_1 z) \exp \mathrm{i}(G_1 y - \Omega t)] \tag{3}$$

appropriate to an incompressible fluid. The boundary condition (2) is then satisfied provided that

$$\phi_0 = \frac{\mathrm{i}\Omega n_0 e E_z}{\rho\Omega^2 - \alpha G_1^3}. \tag{4}$$

The corresponding displacement of the surface, $\zeta$, is then given by

$$\zeta = \mathcal{R}[\zeta_0 \exp \mathrm{i}(G_1 y - \Omega t)] \tag{5}$$

where

$$\zeta_0 = \left(-\frac{n_0 e E_z}{\rho G_1}\right) \frac{1}{v_y^2 - v_1^2} \tag{6}$$

and $v_1^2 = \alpha G_1/\rho$. We note the existence of an undamped resonance when the electron drift velocity approaches the phase velocity of the capillary waves of wavenumber $\boldsymbol{G}_1$. This is the Bragg–Çerenkov scattering in our model. The value of $\zeta_0$ is essentially the depth of the dimple, and it will determine the effective mass of an electron; this mass will therefore increase with increasing electron velocity, tending to infinity at $v_y = v_1$. The electron velocity cannot therefore exceed $v_1$, a result that is required to explain the experimental results in region II, and which was discussed from a slightly different point of view in [6]. The divergence at $v_y = v_1$ is of course unphysical, because we have neglected damping. Damping will arise from two effects: the natural damping of the capillary waves; and radiative loss of capillary wave energy from an electron crystal of effectively finite size. An effectively finite size may arise from the lack of long-range order in the electron crystal, either from its being polycrystalline or because of the inherent lack of long-range crystalline order in two dimensions; or it may arise because only limited areas of the crystal can satisfy the condition for the Bragg–Çerenkov scattering

at any one time, as must be the case with the Corbino geometry. In the presence of damping equation (6) must be modified; the simplest modification that probably describes the essential physics is

$$\zeta_0 = \left(-\frac{n_0 e E_z}{\rho G_1}\right) \frac{1}{v_y^2 - v_1^2 + \mathrm{i} v_d v_y} \tag{7}$$

where $v_d$ is a measure of the strength of the damping, and where we shall assume that $v_d$ is a constant, independent of $v_y$, when $v_y$ is close to $v_1$.

The displacement of the surface of the helium is then given by

$$-\frac{\rho G_1}{n_0 e E_z}\zeta = \frac{(v_y^2 - v_1^2)}{(v_y^2 - v_1^2)^2 + v_d^2 v_y^2} \cos(G_1 y - \Omega t) + \frac{v_d v_y}{(v_y^2 - v_1^2)^2 + v_d^2 v_y^2} \sin(G_1 y - \Omega t). \tag{8}$$

Consider the electron at the point $y = \Omega t / G_1$. Interaction with the distorted surface of the helium will lead to a horizontal force on the electron, which is given by

$$F = e E_z \left(\frac{\partial \zeta}{\partial y}\right)_{y=\Omega t/G_1} \tag{9}$$

as we see if we remember that the force acting on the electron from the helium must be normal to the helium surface. It follows that

$$F = -\frac{n_0 e^2 E_z^2}{\rho} \frac{v_d v_y}{(v_y^2 - v_1^2)^2 + v_d^2 v_y^2}. \tag{10}$$

As a function of the velocity $v_y$ this force has a maximum magnitude given by

$$F_{\max} = \frac{n_0 e^2 E_z^2}{\rho v_d v_1} \tag{11}$$

if the damping is small. An electron will be subject to an electromagnetic force in the $y$-direction equal to $-e v_x B$. In a steady state this force will be balanced by a combination of the force (10) and the ordinary drag force on the electron due to the scattering of thermal ripplons. The rapidly increasing value of (10) as $v_y \to v_1$ leads to the electron velocity becoming constant at a value close to $v_1$, and hence to the behaviour observed in region II, as suggested in [5, 6]. However, as the electromagnetic force increases, there comes a point when the difference between the electromagnetic force and the ordinary drag force exceeds $F_{\max}$, and the balance of forces is no longer possible. At this point the electrons must decouple from the dimple lattice. The dimple lattice must disappear, the electron velocity $v_y$ must become significantly greater than $v_1$ and the electron conductivity must become similar to that in the fluid phase, as is observed in region III.

The idea that region III is associated with sliding and destruction of the dimple lattice was suggested by Shirahama and Kono [4]. However, the details of their picture differ from those suggested here. In their treatment they assume for the purposes of calculating the threshold field for sliding that the deformation of the helium surface is rigid and static. The treatment we give here takes proper account of the dynamical response of the helium surface.

The critical value of the conductivity $\sigma_{xx}$ at which sliding starts to occur is easily shown to be given by

$$\sigma_c = \frac{n_0 F_{\max}}{B^2 v_1} = \frac{n_0^2 e^2 E_z^2}{\alpha G_1 B^2 v_d}. \tag{12}$$

Comparison of the details with experiment is difficult for two reasons. First, the Corbino geometry implies that the electron current density is not spatially uniform, and that the relative directions of these currents and the reciprocal lattice vectors of the crystal vary from place

to place. The latter problem will be intensified if the electrons do not form a single crystal. Secondly, the damping coefficient, $v_d$, is difficult to calculate, especially under spatially non-uniform conditions. Only limited areas of the crystal can be involved at any one time in the Bragg–Çerenkov scattering. The damping will depend in part on the extent to which these regions radiate the generated capillary waves into other parts of the crystal, which we have not yet attempted to calculate. Indeed, it is not yet clear how the damping might depend on the detailed conditions, such as the density $n_0$.

## 3. Conclusions

In summary, we have presented calculations based on a simple classical model to help us understand *in a unified way* the observed non-linear magnetoconductivity of the surface electrons on liquid helium when the electrons are in the crystal phase. The model includes the possibility of Bragg–Çerenkov scattering of capillary waves, first discussed in quantum terms by Dykman and Rubo [6], and it shows how decoupling of the crystal from the underlying dimple lattice can occur at high fields. The decoupling is shown to occur in a way that is significantly different from that suggested by Shirahama and Kono [4].

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